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DEVELOPMENT OF ANALYTICAL ORBIT PROPAGATION TECHNIQUE WITH DRAG

FINAL REPORT

CONTRACT NAS9-15445

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I. INTRODUCTION

This Final Report covers the work done by ACM under Contract NAS9-15445 during the time period November 1, 1977 to March 1, 1979. This report will serve as an overview and summary of the work done. More details on the work are available from the individual reports listed in the references.

1.0 Orbit Computation Methods

The methods for computing satellite orbits can generally be put into two classifications:

(1) *Numerical Methods* - The solution to the satellite differential equations are solved in a step-by-step manner using a mathematical algorithm taken from numerical analysis. Very precise orbits can be computed in this way. Detailed models of the perturbing forces are allowed. An important disadvantage is that these methods usually require significant amounts of computer execution time. Hence they are impractical for many applications where rapid calculations are required. These methods are also referred to as *special perturbations*.

(2) *Analytical Methods* - The solution is expressed by explicit functions of the independent variable. Evaluation of these functions yield the state vector of the satellite at the desired time. No intermediate calculations are necessary. Accordingly, rapid generation of satellite ephemerides is possible. These methods are also referred to as *general perturbations*.

The various available methods have been applied to astrodynamics problems, according to their particular advantages. However, analytical methods have not found wide acceptance because they usually did not contain sufficiently precise models of the perturbing forces. Also, a large initial investment in manpower is required in order to develop the explicit functions. These disadvantages have been overcome in recent years due to advances in celestial mechanics. The purpose of the work described in this report was to apply an up-to-date analytical orbit calculation method in the development of an efficient and accurate orbit prediction program.

2.0 Applications of Analytical Methods

With the development of NASA's Space Transportation System, and in particular the Shuttle Orbiter, new demands are being made on computer software for orbit computation. The emphasis is on rapid calculations in order to support the many planned STS flights. Also, the increased complexity of STS flights means that the overall computation load will be increased, requiring that the most efficient mathematical algorithms be used in the software. For these reasons, analytical methods are preferred for many applications in support of Shuttle missions. They are particularly applicable to iterative calculations such as targeting routines and orbit determination routines, and in other cases where time critical orbit calculations are required.

II. BACKGROUND AND SCOPE OF THE WORK

1.0 Background

A fundamental new idea in celestial mechanics was presented by Dr. Gerhard Scheifele in 1970. He expressed the satellite equations of motion in an extended phase space, using the true or eccentric anomaly as the independent variable. Since the formulation was canonical, it found direct application to analytical satellite theories.

A set of orbital elements similar to Delaunay's elements (called DS-elements) were developed by making use of the new formalism. It was found that these elements presented significant advantages in developing analytical theories. A solution to the J_2 oblateness problem was carried out, and it was shown that this solution was more accurate than solutions based on classical theories.

Singularities at zero eccentricity and inclination were eliminated by the development of yet another set of extended-space canonical elements: the Poincaré-Similar elements in true anomaly, called PSϕ-elements. The J_2 solution in true anomaly DS-elements was converted to PSϕ-elements.

Major advances to the realization of a completely analytical solution to the near-earth satellite problem were made by ACM under NASA/JSC Contract NAS9-15171. For the first time, realistic models of the perturbing forces were included in the solution. The solution of a drag-perturbed satellite was carried out, using an atmospheric density model that includes all the important periodic variations in density. In addition, the implementation of this model into the analytical solution allows for coupling between drag and J_2, and between drag and itself. The long period effects due to the zonal geopotential terms were also added to the solution. An arbitrary number of zonal terms is allowed.

Also, under Contract NAS9-15171, the first steps were made toward the conversion of the analytical solution into efficient computer software. A modular structure was developed in order to allow maximum flexibility in applications. The short period terms of the J_2 solution were included in the software, that was called the Analytical Satellite Orbit Prediction Program (ASOP).

2.0 Scope

Under Contract NAS9-15445, further developments were carried out on the ASOP computer program. (The previous version of ASOP contained only the short period J_2 terms.) Particular emphasis was given to building a modular program structure. The work can be divided into the following major parts:

(1) *Further Mathematical Development* - Analytical initialization formulas were developed for including the effects of the tesseral (time dependent) geopotential terms. Further refinements were made on the drag solution, with the goal of establishing a concise formulation.

(2) *Development of Program Structure* - The modular program structure was developed, including module interfaces and data management.

(3) *Development of Program Modules* - Modules were developed for (a) atmospheric drag, (b) tesseral geopotential terms, (c) J_2 short period and first order secular terms, (d) J_2 second order and long period terms, and (e) second order secular and long period terms due to higher order zonals. The computer code of each module was optimized with respect to computer storage.

(4) *Program Documentation* - Program documentation and user's guide was prepared for ASOP.

(5) *Program Test and Verification* - Individual modules and options were tested. The combined program was verified by comparing to a precision numerical integration.

All modules that were developed in this work make use of the PSφ- elements that were developed by ACM under previous NASA contracts. These are Poincaré-Similar elements based on the true anomaly as independent variable. Therefore, the ASOP program contains no singularities for zero eccentricity or inclination. The modular structure of ASOP allows the user to build an analytical orbit prediction program that is consistent with his computer memory restrictions and accuracy requirements. The ASOP program documentation is given in Reference 1.

III. SUMMARY OF THE INDIVIDUAL TASKS

1.0 Analytic Drag Module

A set of subroutines was developed that gives the atmospheric drag effects on a satellite orbit. They were implemented into ASOP.

The atmospheric density model and drag solution were taken from the analytical developments carried out under Contract NAS9-15171. The module code was compacted and optimized with respect to computer runtime and storage. More details on the work are given in References 1 and 2.

2.0 Tesseral Terms Initialization Module

The dominant perturbations of the motion of a satellite near the earth are due to the non-symmetrical gravitational field and the atmospheric drag. The gravitational field may be divided in two classes: terms independent of time (zonal harmonics) and terms which depend explicitly on time (tesseral harmonics). This task concerns the unified treatment of the tesseral terms in the PSϕ theory and their implementation into ASOP.

The perturbations due to the tesseral harmonics can be placed in four categories:

(1) Short period perturbations with a magnitude of about J_2^2 .

(2) Intermediate period perturbations with a magnitude of between J_2 and J_2^2 .

(3) Along track secular perturbations induced by the periodic perturbations in the mean motion.

(4) Resonant perturbations.

For near earth satellites, the atmospheric drag perturbation continually pulls the satellite in and out of the different

long period resonant frequencies. The result is that the resonances never become apparent and may be neglected.

Since the J_2 theory has been developed only to first order, the second order short period tesseral perturbations may be neglected. For the same reason, the intermediate period perturbations should be included if first order accuracy is to be maintained. The tesseral harmonics have no true secular perturbation but the periodicities in the mean motion induce a secular perturbation in the mean anomaly. This secular perturbation may be determined by simply using the average mean motion instead of the osculating mean motion. Graf[†] finds the average mean motion in a numerical manner. The numerical studies with that approach verify the assumption that use of the average mean motion does account for the apparent secular trend in the mean anomaly. Although the results are good, use of a numerical method would be inconsistent with the idea of having a completely analytical theory.

To complete the solution of the motion of a near earth satellite, the averaged mean motion and the intermediate period perturbations were found in a completely analytical manner.

Since the previous PSϕ theory applied Von Zeipel's solution technique, it seemed natural to return to this method for a solution of the tesseral perturbation. As in the previous developments, the solution was first developed in DSϕ-elements, then converted to PSϕ- elements to avoid singularities. Details of the work are given in Reference 3. Numerical testing of this module is described in Reference 4.

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† See Final Report on Contract NAS9-15171.

3.0 Second Order Secular and Long Period Terms Module

The second order secular and long period terms due to all zonal harmonics were implemented into ASOP. The analytical formulation was taken from the work done under Contract NAS9-15171.

This module contains two parts:

(1) Terms due to J_2^2 and,

(2) Terms due to all necessary higher order zonal terms.

(The work done in building this module covered two tasks in Contract NAS9-15445 Statement of Work). Documentation of the equations in this module are given in Reference 5. A report on the testout of this module is given in Reference 6.

4.0 Verification Testing of the ASOP Program

The accuracy of each analytical theory implemented in the ASOP program was determined during the development of the model. However, a final checkout was required to further determine the accuracy and the limits of the whole program after all the models had been included. Therefore, a series of tests were designed in order to evaluate the total program. Twenty-five (25) test cases were run on a UNIVAC 1110 system. A detailed description of test cases and discussion of results are given in Reference 7. The analytical solutions obtained from ASOP were compared to numerical solutions obtained from the KSFAST program.

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